Written Exam at the Department of Economics winter 2020-21 R

Financial Theory and Models

24 February 2021

(3½-hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 3 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

Problem 1. Consider a stock in a binary one-period model with value 100 USD at time $t = 0$. We assume that the value increases by 25 percent in the up-state and decreases with 15 percent in the down-state. The risk free interest rate $r = 5$ percent.

(i) Calculate the risk neutral probability distribution at maturity.

(ii) Calculate the arbitrage free price c of a European call option written on this stock with strike price $K = 75$ USD.

Problem 2. Particles in the cosmic radiation with energies exceeding 10^9 eV (one giga electron-volt) reach the surface of the Earth at a rate of approximately 10,000 particles per square meter per second and may cause damage to electronic circuitry and computers resulting in unexpected data errors.

It is believed that these particles arrive independently and at random, and that a fixed small fraction of them lead to computer errors. We consider the random variable X that counts the number of data errors per year in a certain computer caused by cosmic radiation.

(i) What is the distribution of X in the model described above?

Suppose that the mean $E(X) = 2$.

(ii) What is the probability that four or more incidents of computer failure caused by cosmic radiation occur in the computer under consideration during a one year period?

(iii) What is the distribution of the waiting time between computer failures in the computer under consideration, and what is the mean?

Problem 3. Consider the Ito process

$$X_t = B_t \qquad B_0 = 0.$$

Show that

$$\int_0^t B_s \, dB_s = \frac{1}{2} B_t^2 - \frac{t}{2} \, .$$

Hint: Apply Ito's lemma to the function $g(t, x) = \frac{1}{2}x^2$.

Problem 4. Assume that the short rate r_t beginning in $r_0 > 0$ is driven by the stochastic differential equation

$$dr_t = e^{-at} \, dt + \sigma \, dB_t \tag{1}$$

where B_t is the Brownian motion, and a, σ are positive constants.

(i) Integrate (1) to obtain a formula for r_t .

(ii) Calculate the mean $E[r_t]$ and variance $\mathrm{Var}[r_t]$ for $t \geq 0$.

(iii) Find the limit of $E[r_t]$ for $t \to \infty$.

(iv) Why is this a bad model for the term structure?